Number Strings

- Students use number relationships to solve problems and to learn number facts.
- Students use known facts and relationships to determine unknown facts.
- Students develop and test conjectures.
- Students make generalizations about mathematical relationships, operations and properties.

This routine focuses on developing a sense of pattern and relationships among related problems. The task is at a higher level than merely recalling basic facts. Students identify and describe number patterns and relationships within and among equations. Students make conjectures about the patterns and relationships they notice. During this process, students explain their reasoning. Over time, students develop generalizations about important number relationships, operations and properties. These generalizations assist in solving problems and learning number facts.

Materials:

- Prepared list of number strings
- Whiteboard, chart paper, or overhead transparency
- Student journals, whiteboards, or scratch paper

Time:

15 minutes per session

Directions:

Example:

Grade 3	Grade 4	Grade 5	Middle Grades
a. 3 x 2 =	a. 2 x 5 =	a. 2 x 5 =	a. 2 x 5 =
b. 3 x 4 =	b. 4 x 5 =	b. 4 x 5 =	b. 4 x 5 =
c. 3 x 8 =	c. 8 x 5 =	c. 8 x 5 =	c. 8 x 5 =
d. 3 x 16 =	d. 16 x 5 =	d. 16 x 5 =	d. 16 x 5 =
e. 3 x 32 =	e. 32 x 5 =	e. 32 x 5 =	e. 32 x 5 =
f. 3 x 64 =	f. 48 x 50 =	f. 48 x 50 =	f. 48 x 50 =
		g. 48 x 0.5 =	g. 48 x 500 =
			h. 48 x 0.5 =
			i. 48 x ½ =
			j. 48 x ¼ =

1. Write equation "a" and ask students to solve mentally (e.g. 2 x 5 =).
   Equation "a" should be easily accessible to all students.
2. Have students check their answer with a partner.
3. Ask one student to share his/her solution with the class. Write the answer on the board to complete the equation (2 x 5 = 10).
4. Students show thumbs up or thumbs down for agreement or disagreement.
   - If there is agreement, go to equation "b."
   - If there is disagreement, facilitate a class conversation around the strategies the student(s) used to arrive at the answer. Allow students to revise answers.
5. Give the students problem "b" to solve mentally (e.g., 4 x 5). Repeat, #'s 2, 3, and 4 above.
6. Write problem "c" (e.g., 8 x 5). Ask students how they could use what they know about the first two equations to solve this equation. Partner talk.
7. A volunteer shares his/her mathematical reasoning that was used to derive an answer to this equation.

   Note: If students are having difficulty sharing relationships, ask questions such as the following:
   - *How are equations "a" and "b" alike?*
   - *How are equations "a" and "b" different?*
   - *Describe the relationship between the factors.*
   - *Describe the relationship between the products.*
   - *How can we use these relationships to predict the product for equation "c?"*
8. Write problem "d" (e.g., 16 x 5). Ask students to predict their answer to this problem. Students share their predictions with their partner and explain their thinking. Teacher writes predictions on the board.
9. A volunteer shares his/her mathematical reasoning that derived the answer to this equation.
10. Repeat steps 8 and 9 for equations "e," "f," "g," and "h."

    Note: When students get to an equation that does not necessarily follow the same pattern (e.g., doubling), the discussion should yield many different strategies. (e.g., "48 x 5 = " could be solved by adding the products of 16 x 5 and 32 x 5, or by multiplying the product of 8 x 5 by 6, or by multiplying the product of 2 x 5 by 24, etc.)
11. When the string is completed, facilitate a conversation about how relating a known equation can help students solve unknown equations. Listen for what relationships students notice throughout the string and how students are able to extend patterns beyond the string you have written. Ask students to make statements about the patterns and/or relationships that helped them to complete the string.
12. Examine the "conjectures" that the students share. Ask questions such as:
    - *Will doubling one factor always result in a doubled product? How can you prove your conjecture?*
    - *Will this always work? How can you prove your conjecture?*

Generalizations to Develop Through Number Strings

NOTE:

Do not tell students these generalizations. Ask students to make conjectures first and then ask them to test their conjectures using three or more examples. If the conjectures always hold true, then the students can make "generalizations.

Generalization 1:

- In multiplication, many strings begin by doubling one factor while leaving the other factor the same (e.g. 2 x 5; 4 x 5). **By whatever amount the factor is multiplied, the product will be multiplied by the same amount** (e.g., 2 x 5 = 10; 4 x 5 = 20. e.g., 3 x 2 = 6; 3 x 4 = 12).
- In division, this relationship holds true with the **dividend** and the quotient. **As the dividend is doubled, the quotient is doubled accordingly** (e.g., 8 ÷ 2 = 4; 16 ÷ 2 = 8).
- In division, the **divisor** has an inverse (opposite) relationship with the quotient. **As the divisor is multiplied by an amount, the quotient is divided by that same amount** (e.g., 36 ÷ 3 = 12; 36 ÷ 6 = 6).

Generalization 2

- In multiplication strings, sometimes, the pattern is predictable because a factor is being doubled over and over, and the product doubles, but then, the pattern may change and the numbers and products cease to double (e.g., 8 x 5 = 40; 16 x 5 = 80; 32 x 5 = 160; then, 48 x 5 = ?).
- In order to make sense of this situation, a student must understand the associated Big Idea: **The Distributive Property of Multiplication. When a number is being multiplied by a particular factor, it is equivalent to multiplying the number by the parts that make up that factor.** For example:

48 = 16 + 32
48 x 5 = (16 x 5) + (32 + 5)

3 x 12 = (3 x 8) + (3 x 4); or
3 x 12 = (3 x 10) + (3 x 2); or
3 x 12 = (3 x 6) + (3 x 6); etc.

- The **Distributive Property** also states that **when a dividend is being divided by a particular divisor, it is equivalent to dividing the parts that make up that dividend by the same divisor and then adding the quotients.** For example:

2814 = 2800 + 14
2814 ÷ 14 = (2800 ÷ 14) + (14 ÷ 14)
2814 ÷ 14 = 200 + 1 = 201

Students generally find it easy to find 2800 ÷ 14 = 200 and 14 ÷ 14 = 1.
If they add 200 + 1, students have the answer.

Using Strings to Learn Multiplication Facts

Strings can be helpful to assist students to learn their multiplication facts as they learn to see the relationships among the facts.

Example I: If a student cannot remember 8 x 6, but knows 4 x 6, all the student has to do is double the product of 4 x 6 because 8 = 2 x 4.

4 x 6 = 24
8 x 6 = 48

Example II: If a student cannot remember 8 x 6, but knows 2 x 6 and 6 x 6, all the student has to do is find the product of these two equations and then find the sum of the products because 8 = 2 + 6.

2 x 6 = 12 6 x 6 = 36 8 x 6 = 48

Difficult Multiplication Facts: If a student has difficulty remembering some facts such as 7 x 8, all the student has to do is use what he or she knows. 7 = 2 + 5. Students generally do not have difficulty with their 2's or 5's.

7 x 8 = ?
7 = 2 + 5
2 x 8 = 16
5 x 8 = 40
16 + 40 = 56
7 x 8 = 56

Guiding Questions:

- What patterns do you see?
- What stayed the same?
- What changed?
- How did it change?
- How did the first equation help you to figure out the answer to the next equation?
- Does this always work? How do you know?
- How are equations "a" and "b" alike?
- How are equations "a" and "b" different?
- What is the relationship between the factors?
- What is the relationship between the products?
- How can we use these relationships to predict the product for equation "c?"
- What is the relationship between the dividends?
- What is the relationship between the divisors?
- How can we use these relationships to predict the quotient for equation "c?"
- What is the relationship between the quotients?

Scaffold:

Begin with strings that grow in a predictable way and are easily accessible to all students

Possible Number Strings:

2 x 5 =
4 x 5 =
8 x 5 =
16 x 5 =
32 x 5 =
48 x 5 =
48 x 50 =
48 x 0.5 =
48 x 0.25

1 x 10 =
2 x 10 =
3 x 10 =
6 x 10 =
6 x 20 =
6 x 200 =
6 x 0.2 =
6 x 0.02 =

1 x 12 =
2 x 12 =
3 x 12 =
6 x 12 =
8 x 12 =
8 x 1.2 =
8 x 120 =
8 x 121 =

8 x 2 =
8 x 3 =
8 x 4 =
8 x 8 =
8 x 10 =
4 x 10 =
12 x 10 =
1.2 x 10 =

12 ÷ 12 =
12 ÷ 6 =
12 ÷ 4 =
12 ÷ 3 =
12 ÷ 2 =
12 ÷ 1 =
12 ÷ 1/2 =
12 ÷ 1/4 =

36 ÷ 3 =
36 ÷ 6 =
18 ÷ 6 =
180 ÷ 6 =
180 ÷ 12 =
1800 ÷ 12 =
3600 ÷ 12 =
3.6 ÷ 12 =

8 ÷ 2 =
16 ÷ 2 =
32 ÷ 2 =
48 ÷ 2 =
48 ÷ 4 =
480 ÷ 4 =
484 ÷ 4 =
480 ÷ 40 =

14 ÷ 7 =
140 ÷ 7 =
280 ÷ 7 =
287 ÷ 7 =
280 ÷ 14 =
0.28 ÷ 14 =
2800 ÷ 14 =
2814 ÷ 14 =

2 x 6 =
20 x 6 =
200 x 6 =
0.2 x 6 =
0.02 x 6 =
0.002 x 6 =

3 x 7 =
30 x 7 =
0.3 x 7 =
0.3 x 0. 7 =
0.03 x 0.7 =
0.03 x 0.07 =

15 ÷ 5 =
15 ÷ 0.5 =
15 ÷ 0.05 =
15 ÷ 0.005 =
15 ÷ 50 =
15 ÷ 500 =

12 ÷ 3 =
1.2 ÷ 3 =
0.12 ÷ 3 =
0.012 ÷ 3 =
12 ÷ 30 =
12 ÷ 300 =